ISSN 0038-0946, Solar System Research, 2007, Vol. 41, No. 5, pp. 413–419. © Pleiades Publishing, Inc., 2007.
Original Russian Text © V.A. Avdyushev, M.A. Banshchikova, 2007, published in Astronomicheskii Vestnik, 2007, Vol. 41, No. 5, pp. 446–452.

Regions of Possible Motions for New Jovian Satellites

V. A. Avdyushev and M. A. Banshchikova

Research Institute of Applied Mathematics and Mechanics, Tomsk State University, Tomsk, Russia

Received October 26, 2006; in final form, February 16, 2007

Abstract—We present the results of our simulation and study of the regions of possible motions for 46 newly discovered Jovian satellites. We show that the orbits of some satellites (such as S/2003 J02, S/2003 J03, S/2003 J04, S/2003 J10, S/2003 J12, and S/2003 J23) presently cannot yet be determined with an acceptable accuracy for planning observations, because the amount of observational information is insufficient.

PACS: 95.10.Ce,96.30.L-,95.10.Eg.91.10.Sp

DOI: 10.1134/S0038094607050048

INTRODUCTION

From 1999 to 2003, several groups of astronomers from the Arizona, Hawaii, and Cambridge Universities discovered 46 new Jovian satellites (http://cfa-www.harvard.edu/iau/mpc.html; http://cfa-www.harvard.edu/iau/cbat.html). All satellites are distant and have been arbitrarily divided into six groups: Themisto, Himalia, Carpo (prograde motion) and Ananke, Carme, Pasithee (retrograde motion). The satellite orbits are irregular (with high eccentricities and inclinations) and, being far from the planet, are strongly perturbed by the Sun.

In 2002, Marsden and Jacobson (Sheppard et al., 2002) presented orbital parameters for the first 11 discovered satellites that were improved from the then available observations; and only in 2005 was the paper by Emelyanov (2005) published where the orbits for all of the satellites known to date were determined. In their papers, the authors resort to the standard least-squares method (LSM) to improve orbital parameters from available observations; the quality of orbital parameters is traditionally estimated from the root-mean-square (rms) error obtained from the residuals of the calculated and observed positions for a celestial body.

In general, however, the rms error cannot be a sufficient characteristic of the quality, notably for the satellites for which the times of observations are distributed in a short interval and the observations themselves cover only a short orbital arc.

The point is that the closeness of the true and calculated orbital parameters is determined in such problems not so much by the rms error as by the corresponding covariance matrix. The latter depends not only on the rms error, but also, mainly, on peculiarities in the distribution of observations.

Therefore, for a more intensive study of the quality of LSM estimates, one usually resorts to simulating the so-called regions of possible motions (Milani, 1999; Bordovitsyna et al., 2001; Muinonen et al., 2006), where not one LSM orbit, but the whole family of the most probable orbits constructed using the covariance error matrix is investigated.

In this paper, we present the results of our simulation and study of the regions of possible motions for all new Jovian satellites.

ORBIT DETERMINATION

We considered the motion of satellites in the gravitational field of Jupiter under the influence of attraction from the Sun, the giant planets, and the Galilean satellites. The satellite orbits were simulated numerically based on equations of motion in rectangular coordinates (Banshchikova and Avdyushev, 2006), which were integrated by Everhart's method (Everhart, 1974; Avdyushev, 2006) with a coordinate accuracy of 10^{-12} AU.

As the parameters to be estimated, we took the rectangular coordinates $\mathbf{x}_0$ and velocities $\dot{\mathbf{x}}_0$ at an initial epoch t_0. These were predetermined from observations by the Laplace method. We improved the initial dynamical state vector $\mathbf{q}_0 = (\mathbf{x}_0, \dot{\mathbf{x}}_0)$ based on all of the available ground-based CCD observations (http://lnfm1.sai.msu.ru/neb/nss/index.html) using the least-squares method. The isochronous derivatives needed for orbit correction were calculated by numerically integrating the variational equations together with the equations of motion.

The determination of the orbital parameters $\mathbf{q}_0$ formally consisted in minimizing the rms value of σ^2 obtained from the residuals of the model $\mathbf{p}_C = \mathbf{p}_C(\mathbf{q}_0)$ relative to the observational data $\mathbf{p}_O$ at the corresponding times of observations. Algorithmically, this process was reduced to solving the system of so-called conditional equations by the least-squares method: $\mathbf{p}_O - \mathbf{p}_C = \mathbf{J}\Delta\mathbf{q}_0$, where ~~$\mathbf{J}$ =~~ $\mathbf{J} = \partial\mathbf{p}_C/\partial\mathbf{q}_0$ is the matrix of conditional equations and $\Delta\mathbf{q}_0$ are the corrections to $\mathbf{q}_0$.

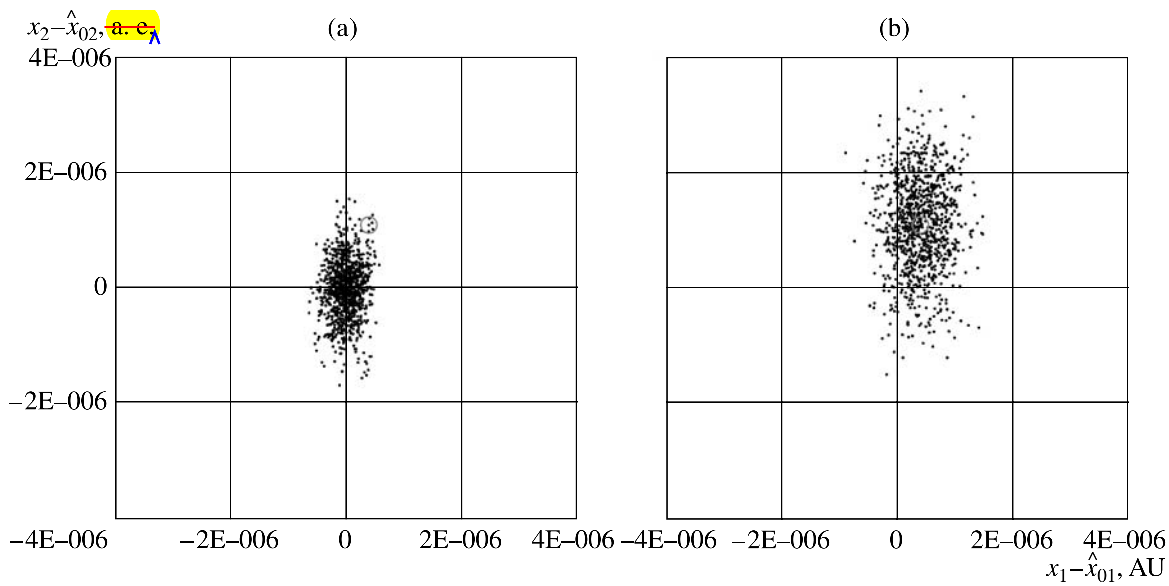

Fig. 1. (a) Distribution of LSM solutions for various samples of model observations (Themisto). (b) Probabilistic region constructed for one of the LSM solutions (at the center of the gray ring) from the covariance matrix (Themisto).

After the orbit determination for each satellite, we obtained the covariance matrices (Elyasberg, 1976; Montenbruck and Gill, 2005)

$$\mathbf{C}_0 = \sigma_0^2 (\mathbf{J}^T\mathbf{J})^{-1} \quad (\sigma_0^2 = \min \sigma^2),$$

which characterize the probabilistic scatter of random errors in the parameters $\mathbf{q}_0$ being estimated. Based on the covariance matrices, we then simulated the initial regions of possible orbital parameters.

SIMULATING THE INITIAL REGION OF POSSIBLE ORBITAL PARAMETERS

We constructed the initial region of possible solutions $\mathbf{q}_0$ for each satellite relative to the LSM estimate $\hat{\mathbf{q}}_0$ using the covariance matrix $\mathbf{C}_0$ from the formula (Bordovitsyna et al., 2001)

$$\mathbf{q}_0^i = \mathbf{A}\boldsymbol{\eta}^i + \hat{\mathbf{q}}_0 \quad (i = 1, \ldots, N), \tag{1}$$

where $\boldsymbol{\eta}^i$ is the six-dimensional vector of normally distributed random numbers, $\mathbf{A}$ is the triangular matrix for which $\mathbf{A}^T\mathbf{A} = \mathbf{C}_0$, and N is the number of solutions under consideration. Note that the decomposition of the covariance matrix $\mathbf{C}_0 = \mathbf{A}^T\mathbf{A}$ is accomplishable and unique, since the latter is symmetric and positively defined[1]. In the phase space of coordinates and velocities, solutions (1) will fill the hyperellipsoid specified by the covariance matrix $\mathbf{C}_0$ (see, e.g., Fig. 1b).

The algorithm described above was tested on model problems for two satellites: Themisto and S/2003 J04. Using real observations of the satellites, we obtained the LSM solutions $\hat{\mathbf{q}}_0$. Subsequently, based on these solutions and considering them as the true ones, we simulated fictitious observations at the real times of observations by introducing into them a normally distributed random error with a dispersion of 0.2″ (the limiting accuracy of CCD observations)[2] ~~into them~~ and obtained a set of LSM solutions from 1000 samples of observations (Figs. 1a and 2a).

Finally, for some arbitrary solutions, we constructed the regions of possible initial parameters using the corresponding covariance matrices (Figs. 1b and 2b)[3]. In all cases, the true solutions (the coordinate origins in the figures) fell into the constructed probabilistic regions. Obviously, this serves as an experimental justification for using the probabilistic regions for a qualitative analysis of the orbits being determined.

NUMERICAL RESULTS

As a result of the orbit improvement, the rms errors σ_0 for all satellites were less than 1″ (Fig. 3). However, despite their smallness, the corresponding regions of possible initial parameters turned out to be very varied not only in size, but also in shape.

For most of the satellites discovered in 2003 (S/2003), the probabilistic regions are fairly large and

[1] See, e.g., Verzhbitskii, 2005.

[2] However, such a choice of the dispersion is of no fundamental importance in this case.

[3] The figures show the regions of solutions only in projection onto the (x_1, x_2) plane. The projections of the regions onto other planes look similar: small compact ones for Themisto and extended cigar-shaped ones for S/2003 J04. Note also that Figs. 1 and 2 and, below, Figs. 4 and 6 are presented in the J2000 equatorial coordinate system.)

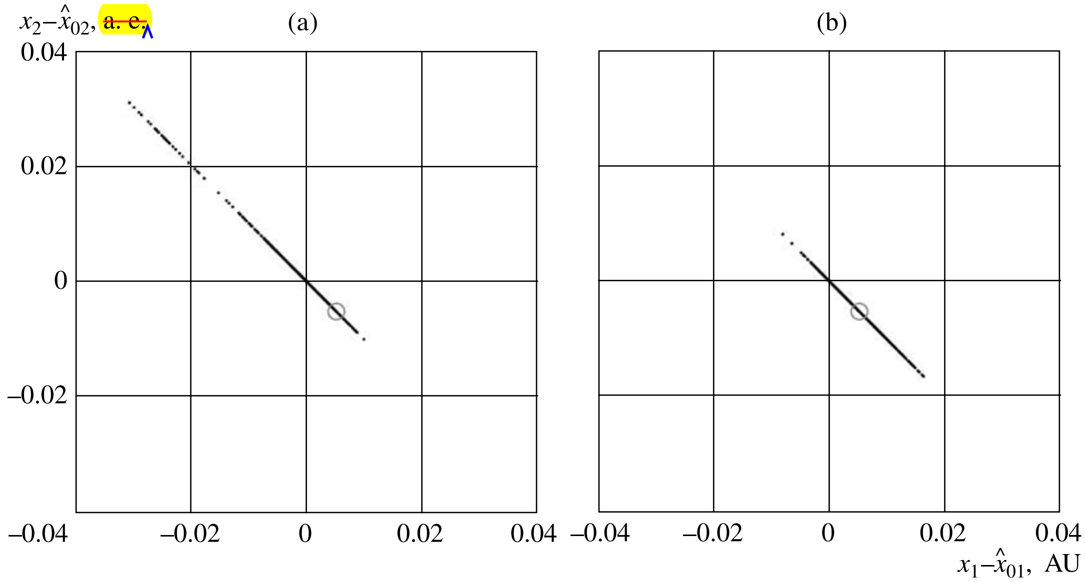

Fig. 2. Same as Fig. 1 for S/2003 J04.

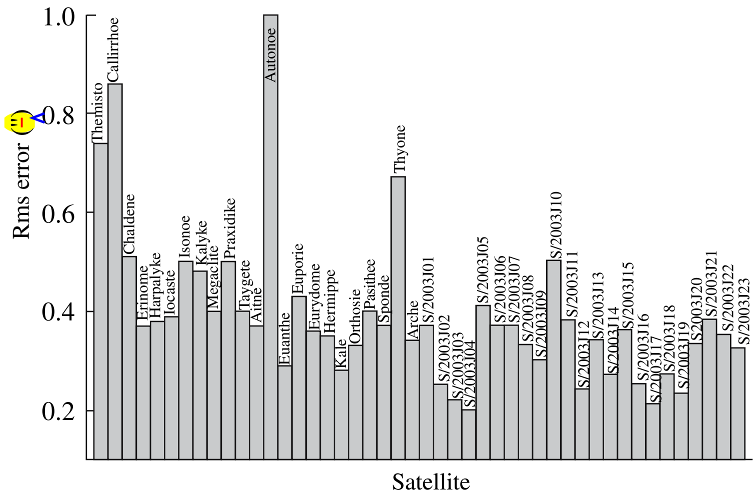

Fig. 3. Rms errors.

strongly elongated (e.g., Fig. 4; S/2003 J04). This is mainly due to the small number of observations, whose times are concentrated in a short interval, up to 100 days (Fig. 5). In such problems, the covariance matrices have large condition numbers and this directly leads to a strong elongation of the probabilistic regions. For the satellites for which the times of observations cover fairly long intervals, the probabilistic regions are considerably smaller (e.g., Fig. 4; Themisto).

The large initial probabilistic regions generally suggest that the observations for the corresponding satellites are not yet enough for a confident prediction of the satellite motion, for example, with the goal of planning observations in the future. As an example, Fig. 6 shows for S/2003 J10 how large the scatter of possible satellite positions $\mathbf{x}_1^i = \mathbf{x}(t_0 + \tau, \mathbf{q}_0^i)$ predicted from $\mathbf{q}_0^i$ can be even after one revolution (τ = 716 days) for a large initial probabilistic region of $\mathbf{q}_0^i$; i.e., in fact, the satellite can be anywhere in a wide neighborhood comparable to the orbit itself.

Incidentally, it is interesting to note that the initial probabilistic region in Fig. 6 is elongated toward the Earth. Therefore, its projection onto the geocentric celestial sphere reveals no extended size of the region. As we see from Fig. 7a, the projected region compactly fits within only 2″. Here, the distribution of possible positions is presented in spherical (α, δ) coordinates

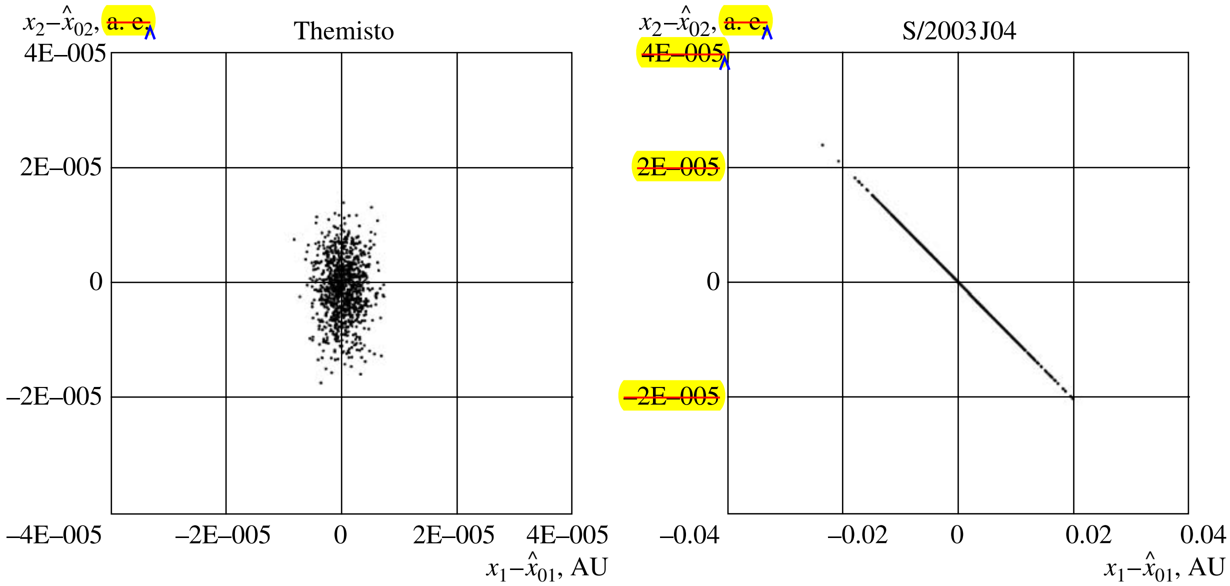

Fig. 4. Probabilistic regions obtained from the covariance matrices based on real satellite observations.

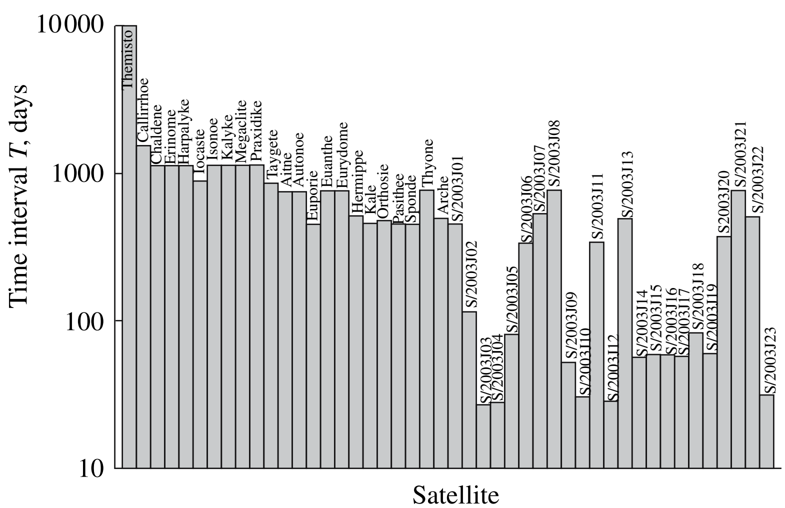

Fig. 5. Time intervals covering the times of all available satellite observations.

(α is the right ascension and δ is the declination) relative to the initial nominal position ($\hat{\alpha}_0$, $\hat{\delta}_0$).

When ground-based observations are planned, the required accuracy of predicting the motion is directly determined by the sizes of the sky field scanned by observational means where the object is expected to appear. For example, if we plan to observe the satellite S/2003 J10 after one revolution, expecting it to appear in a fairly large $1° \times 1°$ field, using the dynamical model of the satellite for its targeting is unacceptable in this case. As Fig. 7b shows, the probabilistic region for S/2003 J10 is so large that much of it falls outside the scanned field centered on the predicted position of the object ($\hat{\alpha}_1$, $\hat{\delta}_1$) and, hence, there is a probability of losing the satellite.

The maximum angular deviation of the possible positions (α^i, δ^i) from the nominal one ($\hat{\alpha}$, $\hat{\delta}$) obtained from LSM estimates may be taken as a characteristic of the size of the probabilistic region in projection onto the celestial sphere:

$$s_{\max} = \max_{i=1,\ldots,N} \sqrt{(\alpha^i - \hat{\alpha})^2 \cos^2\hat{\delta} + (\delta^i - \hat{\delta})^2}.$$

We estimated $s_{\max}$ for each satellite after one revolution at $N = 1000$. The results are shown in Fig. 8, where, as

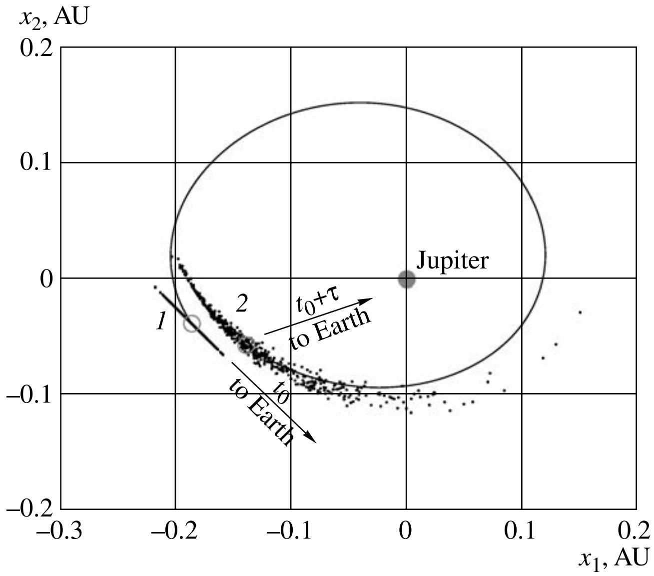

Fig. 6. Probabilistic regions of S/2003 J10 relative to the nominal orbit: 1, at the initial time; 2, after one satellite revolution.

in Figs. 3 and 5, the nomenclature of the satellites is presented in order of their discovery.

As we see from s_{max}, apart from S/2003 J10, there are several more objects (namely, S/2003 J02, S/2003 J03, S/2003 J04, S/2003 J12, and S/2003 J23) that can be lost in attempting to detect them after one revolution in a sky field with angular sizes of ~1°. Naturally, the number of potentially disappearing objects increases as the viewing angle decreases.

It is also important to note that the probabilistic regions increase in size with time (see, e.g., Fig. 6) mainly due to the instability of orbital motion. Therefore, the chances to detect the satellite in the future next time will only decrease as the date of observations recedes from the initial epoch.

Given that objects similar to the new Jovian satellites are observed on telescopes with viewing angles much smaller than 1°, we may assert that, in general, the accuracies of the simulated orbits for the above satellites (S/2003) are unsatisfactory from the standpoint of planning observations.

To increase the accuracy of determining these orbits (in other words, to reduce the regions of possible motions), we must use additional observations that together with the already available ones would cover a long time interval, where possible. Figure 9 shows the correspondence between the sizes of the probabilistic regions at an initial epoch and the spread in times of observations. Here, $|\Delta\mathbf{x}|_{max}$ is the maximum deviation of the possible positions $\mathbf{x}_0^i$ from the nominal one $\hat{\mathbf{x}}_0$:

$$|\Delta\mathbf{x}|_{max} = \max_{i=1,\ldots,N} \left|\mathbf{x}_0^i - \hat{\mathbf{x}}_0\right|;$$

$\bar{T}$ is the dispersion of the times of observations t_i:

$$\bar{T}^2 = \frac{1}{N}\sum_{i=1}^{N}(t_i - \bar{t})^2, \quad \bar{t} = \frac{1}{N}\sum_{i=1}^{N} t_i;$$

and τ is the orbital period of the satellite.

In particular, we see from the figure that small probabilistic regions take place precisely for those satellites

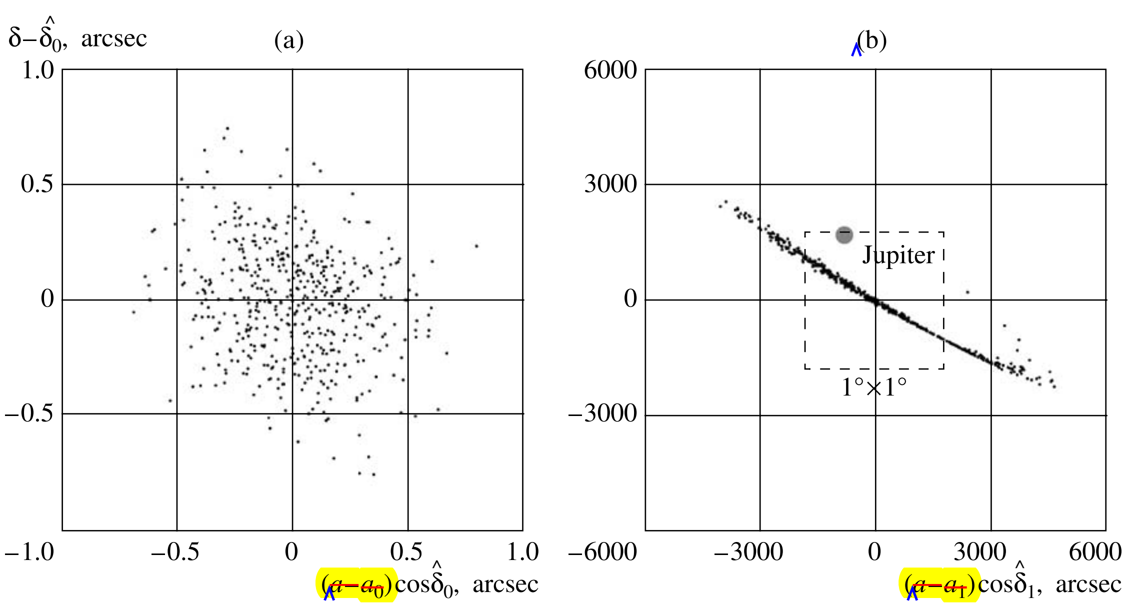

Fig. 7. Same as Fig. 6 in projection onto the geocentric celestial sphere (separately for configurations 1 and 2): (1) at the initial time and (b) after one revolution.

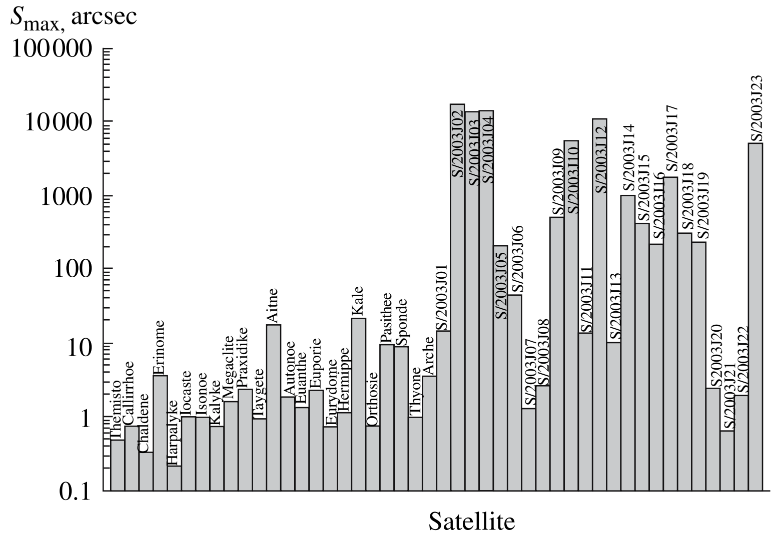

Fig. 8. Maximum angular deviations of the possible satellite positions from the corresponding nominal ones on the geocentric celestial sphere after one revolution.

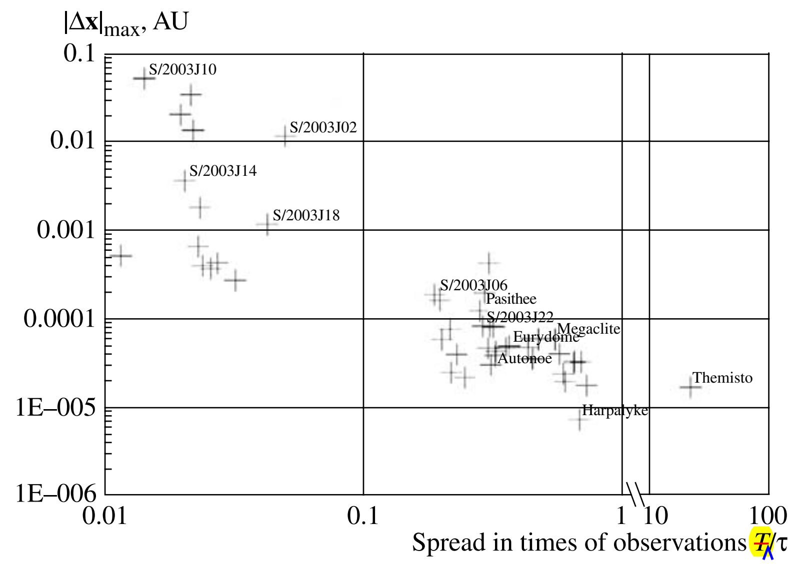

Fig. 9. Correspondence between the sizes of the initial probabilistic regions and the spreads in times of observations.

for which the times of observations are distributed widely. At the same time, for most of the satellites discovered in 2003 and observed in short time intervals, the characteristics $|\Delta \mathbf{x}|_{max}$ and $\bar{T}/\tau$ under consideration correlate weakly. This is mainly because other factors[4] that potentially affect the sizes of the probabilistic region become weighty at a small spread in times of observations. In any case, it can be said with certainty that a long chronology of satellite observations is a firm guarantee for a highly accurate determination of the satellite orbit.

CONCLUSIONS

Thus, we analyzed the probabilistic regions of orbital parameters for new Jovian satellites. We experimentally shown that there are such satellites among them (S/2003 J02, S/2003 J03, S/2003 J04, S/2003 J10, S/2003 J12, and S/2003 J23) whose orbits cannot yet be determined with an acceptable accuracy for planning observations, because the amount of observational

[4] For example, peculiarities in the distribution of observed satellite positions.

information is insufficient. The orbits of these objects are determined with such a large uncertainty that their predicted positions (even after one revolution) can contain large errors comparable to the sizes of the orbits themselves. Such a prediction for pointing a telescope at a satellite (even with a wide scanning field) is actually unreliable, since there is a high probability that the object will just be outside the sky field scanned by the telescope.

We obtained our results in terms of a least-squares problem by numerically simulating the satellite orbits determined from all of the observations available to date.

ACKNOWLEDGMENTS

We are grateful to Prof. A.M. Chernitsov for a discussion of the results and for valuable remarks and suggestions. This work was supported by the Russian Foundation for Basic Research (project no. 05-02-17043).

REFERENCES

Avdyushev, V.A., Gauss–Everhart Integrator. New FORTRAN Code, *Materialy Vseross. konf. "Fundamental'nye i prikladnye problemy sovremennoi mekhaniki"* (Proc. All-Russian Conf. "Fundamental and Applied Problems of Modern Mechanics"), Tomsk, 2006, pp. 413–414.

Ban'shchikova, M.A. and Avdyushev, V.A., Numerical Simulation of Dynamics of Jupiter's Satellites, *Izv. Vyssh. Uchebn. Zaved., Fiz., Prilozh: Nebesn. Mekh. Prikl. Astron.*, 2006, vol. 49, no. 2, pp. 74–82.

Bordovitsyna, T.V., Avdyushev, V.A., and Chernitsov, A.N., New Trends in Numerical Simulation of the Motion of Solar System Small Bodies, *Celest. Mech.*, 2001, vol. 80, nos. 3/4, pp. 227–247.

El'yasberg, P.E., *Opredelenie dvizheniya po rezul'tatam izmerenii* (Motion Definition on the Base of Measurements), Moscow: Nauka, 1976.

Emelyanov, N.V., Ephemerides of the Outer Jovian Satellites, *Astron. Astrophys.*, 2005, vol. 435, no. 3, pp. 1173–1179.

Everhart, E., Implicit Single Sequence Methods for Integrating Orbits, *Celest. Mech.*, 1974, vol. 10, pp. 35–55.

Milani, A., The Identification Problem I: Recovery of Lost Asteroids, *Icarus*, 1999, vol. 137, pp. 269–292.

Montenbruck, O. and Gill, E., *Satellite Orbit. Models, Methods and Applications*, New York: Springer, 2005.

Muinonen, K., Virtanen, J., Granvik, M., and Laakso, T., Asteroid Orbits Using Phase-Space Volumes of Variation, *Mon. Not. R. Astron. Soc.*, 2006, vol. 368, pp. 809–818.

Sheppard, S.S., Jewitt, D.C., Kleyna, J., et al., Satellites of Jupiter, *IAU Circ.*, 2002, no. 7900, p. 1.

Verzhbitskii, V.M., *Chislennye metody. Lineinaya algebra i nelineinye uravneniya* (Numerical Methods. Linear Algebra and Nonlinear Equations), Moscow: ONIKS 21 Vek, 2005.