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DETERMINATION OF THE EFFECTIVE SURFACE OF A SPENT SPACECRAFT TO TAKE INTO ACCOUNT THE INFLUENCE OF LIGHT PRESSURE ON ITS MOTION

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A method for determining the effective surface area of a spent satellite depending on its spatial orientation is suggested on the example of the GLONASS spacecraft (SC). To determine the effective area, a point-set model is considered. The satellite is represented as a set of points uniformly distributed over the SC surface. Then longitude and latitude angles are varied in the system of coordinates rigidly affixed to the SC, and the point set is projected onto the image plane. The contour of the projected set is determined, and its area is calculated. As a result, an approximate dependence of the effective satellite area on two orientation angles is obtained.

Keywords: artificial Earth satellite, spent spacecraft, light pressure, effective spacecraft surface, GLONASS.

INTRODUCTION

At present, the theory of motion of near-Earth objects has been studied in ample details, and many numerical and numerical-analytical models of their motion have been constructed. However, each of these models faces difficulties in determining non-gravitational effects such as light pressure (LP). Taking into account the influence of the LP is one of the basic difficulties in modeling of the orbital motion of operating and spent spacecrafts (SCs) as well as other space debris. The influence of the LP depends on the geometrical and physical properties of the object; therefore, it is impossible to construct a generalized high-precision model, but it is possible to obtain the LP model which will allow one to simulate the orbit with required accuracy for certain types of objects (with known configuration).

At present different approaches are used to take into account the LP influence on satellite dynamics, from purely empirical models of various complexity based on the behavior of the spacecraft in the orbit to the so-called physical models based on preliminary structural analysis of the SC before launching [1]. Our previous studies [2–5] of different LP models on the example of the GLONASS SC have demonstrated the following:

1) the best accuracy is obtained with the model [1] based on physical interaction between the sunlight and the satellite surface with allowance for the SC shape,

2) the application of the standard model of the LP force in which the SC is represented in the form of a sphere leads to inadmissibly large errors in modeling of the orbital motion.

In case of working controllable GLONASS SCs, their orientation in space is easily determined since they are stabilized and their solar panels are perpendicular to the direction toward the Sun, whereas the SC body is oriented toward the Earth (Fig. 1). Hence, the area of the effective SC surface can be easily calculated. By the effective surface we understand that part of the SC surface which interacts directly with the sunlight. Meanwhile, the LP force is directly proportional to the area of the effective surface projection onto the image plane perpendicular to the direction of the LP force, that is, to the average SC cross section.

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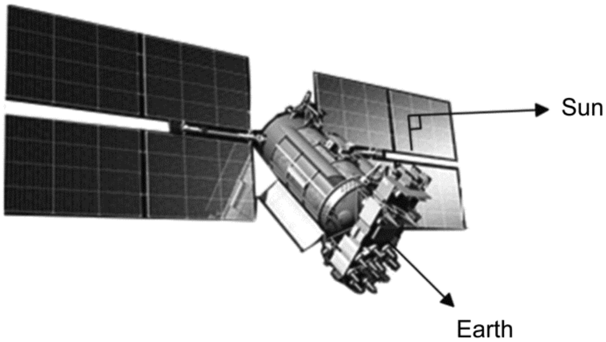

Fig. 1

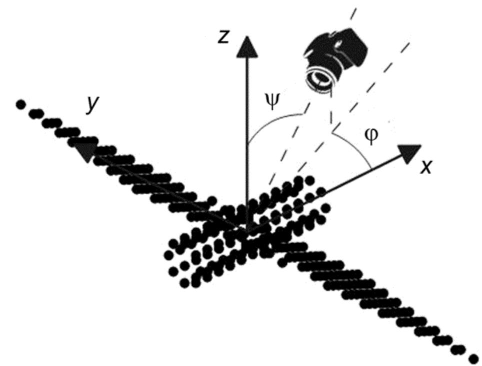

Fig. 2

Fig. 1. GLONASS-M satellite.

Fig. 2. Virtual photo session of the point-set model of the GLONASS satellite.

As to the spent GLONASS SC, the determination of its effective surface or average cross section is complicated by arbitrary spatial SC orientation when its components, including satellite bus, solar panels, or other auxiliary blocks, overlap each other in the image plane, which is not the case for the operating satellites. To overcome this difficulty, we suggest a numerical method for determining the average cross section based on a virtual photo session of point-set model of the SC (Fig. 2).

1. MODELING OF THE EFFECTIVE SC SURFACE

A satellite is represented as a set of points (Fig. 2) uniformly distributed over the SC surface. The reflection coefficient is assigned to each point. Then the virtual photo session is performed: the longitude (φ) and latitude (ψ) angles are varied in the coordinate system rigidly affixed to the SC (Fig. 2), and the set of points is projected onto the image plane (Fig. 3).

The boundary points are determined for each pair φ and ψ using the principle of a rolling ball (Fig. 3) projected onto the plane of the image of the point set. The first boundary point is extreme left. Each subsequent point is determined when the ball touches the plane (the circle with the point in the center of Fig. 3) which envelopes the set rotating counter-clockwise. The ball diameter is chosen so that it cannot fall inside the set.

Grey points in Fig. 3 connected by the grey curve show the boundary of the set of points representing the satellite image. The circle represents the ball that reveals the boundary points. Sorting of points before determination of the subsequent boundary point is established from an increase in the azimuth angle θ: the first point after sorting is designated by unity. The subsequent boundary point is designated by the circle.

Let us describe the main stages of the algorithm for determining the subset of boundary points.

1. We upload the coordinates of points (X_i, Y_i) $(i = 1, \ldots, N)$ of the initial set representing the image of the satellite with the given camera angles into a two-dimensional array. The first subscript designates the number of point i, and the second subscript designates the coordinate X or Y. Here N is the number of points in the set.

2. We determine the extreme left point of the set: by sorting the array elements, we find the coordinates of the pair with the least first coordinate. It assigns the first boundary point $(\overline{X}_1, \overline{Y}_1)$. Above this boundary point, we place the circle (ball) with radius r_{tol} and the center at $(\overline{X}_1, \overline{Y}_1 + r_{\text{tol}})$ (Fig. 3).

3. We sort elements of the array according to the increasing azimuth angle θ with vertex at the current boundary point $(\overline{X}_j, \overline{Y}_j)$ between the fixed direction toward the circle center and directions toward points of the set.

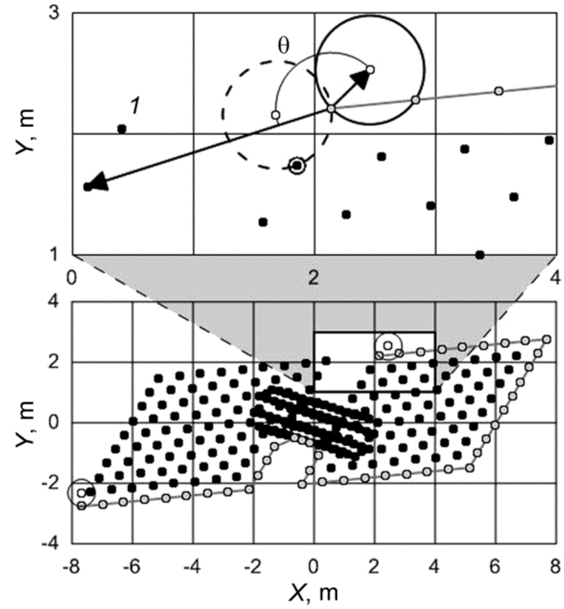

Fig. 3. Search for boundary points. The camera angles were $\varphi = 60°$ and $\psi = 80°$.

4. Passing through the sorted coordinates (X_i, Y_i) $(i = 1, \ldots, N)$, we determine the first pair corresponding to the point whose distance from the current boundary point is less than the admissible value (the diameter of the circle d_{tol}). It is taken as the candidate for the subsequent boundary point $(\bar{X}_{j+1}, \bar{Y}_{j+1})$ (Fig. 3). From its coordinates and the coordinates of the current boundary point, we determine the new center of the circle.

5. We check whether any other point of the set falls within the circle. We continue sorting through the coordinates of pairs from the sorted array and estimate the distance of the corresponding points to the center of the circle. If the distance of a certain point is less than the radius r_{tol}, i.e., if it falls within the circle, we choose it as the candidate for the subsequent boundary point $(\bar{X}_{j+1}, \bar{Y}_{j+1})$. We redefine the center of the circle and repeat item 5. This procedure is carried out until the azimuth angle (with the apex at the current boundary point) between the direction toward already new center of the circle and the point of the set will not become greater than 90°: it makes no sense to consider the remaining points because they will definitely lie beyond the circle.

6. Thus, we determine the subsequent boundary point and go to item 3. The sequence of items 3–5 is executed until the coordinates of the found boundary point coincide with the coordinates of the first point, i.e., until closing of the boundary contour.

Finally, when the boundary points of the set $(\bar{X}_i, \bar{Y}_i)$ $(i = 1, \ldots, M$, where M is the number of boundary points) are known, the area of the average cross section S is expressed as the area of the polygon formed by these points:

$$S = \frac{1}{2}\sum_{i=1}^{M}(\bar{X}_i + \bar{X}_{i+1})(\bar{Y}_i - \bar{Y}_{i+1}) \quad (\bar{X}_{N+1}, \bar{Y}_{N+1}) = (\bar{X}_1, \bar{Y}_1). \tag{1}$$

2. MODELING OF THE SC REFLECTANCE

To estimate the total SC reflectance, pixel images were modeled during virtual photo session. The set of points in the image plane was mapped onto a certain pixel matrix (Fig. 4), and each pixel was assigned the reflection coefficient of the point closest to the image plane inside the pixel. The pixel without points had zero reflection

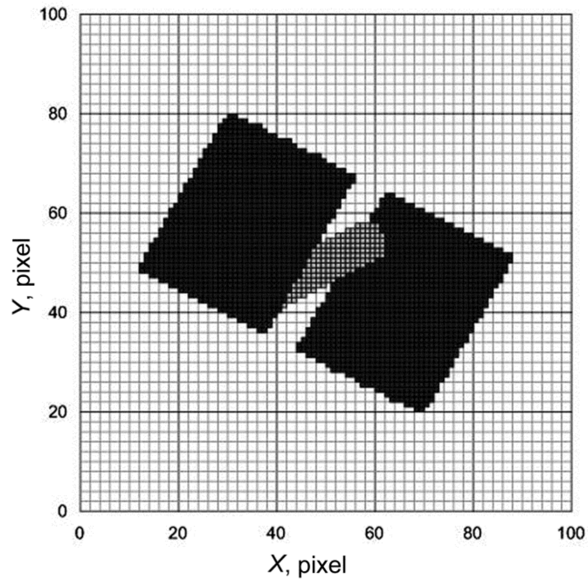

Fig. 4. Pixel image of the model ($N_{\text{pix}} = 100$, $\varphi = 45°$, and $\psi = 120°$).

coefficient. Thus, the total SC reflectance was estimated as the area of the product of the average cross sections by the average reflection coefficient determined as the ratio of the total reflection coefficient to the number of pixels with nonzero coefficients.

The program pixel model is formed as the three-dimensional array

$$D_{ijk} \quad (i,\, j = 1,\, \ldots,\, N_{\text{pix}};\; k = 1,\, 2),$$

where N_{pix} is the number of pixels in the image. The first and second subscripts of the array i and j denote the address of the pixel, and the third subscript denotes the coordinate Z of the point of the pixel closest to the image plane $(k = 1)$ or its reflection coefficient $(k = 2)$. When forming the image array from the initial data array comprising a set of points, the spatial coordinates of points (X_n, Y_n, Z_n) and their reflection coefficients $\sigma_n \quad (n = 1, \ldots, N)$ are alternatively retrieved. For each point, the address of the pixel containing it is determined:

$$i = \left[(N_{\text{pix}} - 1) \frac{X_n + W/2}{W} \right] + 1, \quad j = \left[(N_{\text{pix}} - 1) \frac{Y_n + W/2}{W} \right] + 1;$$

the element D_{ij1} is retrieved and compared with the coordinate Z_n: if $D_{ij1} < Z_n$, then $D_{ij1} := Z_n$ and $D_{ij2} := \sigma_n$. Here the square brackets denote the operation of calculating the integer part of the number, W is the width of the (squared) image expressed in the same units, as coordinates. Before the formation of the array, $D_{ij1} = -W$ and $D_{ij2} = 0 \; (i, j = 1, \ldots, N_{\text{pix}})$. Then the average SC reflectance is written as

$$\sigma = \frac{1}{M} \sum\nolimits_{i,j} D_{ij2}\,, \tag{2}$$

where M is the number of nonempty pixels for which $D_{ij2} \neq 0$. Hence, the total reflectance is

$$R = S\sigma\,. \quad (3)$$

We note, however, that when calculating σ, we can estimate the average cross sectional area from the formula $S = M(W/N_{\text{pix}})^2$; then

$$R = (W/N_{\text{pix}})^2 \sum_{i,j} D_{ij2}\,. \quad (4)$$

3. DETERMINATION OF THE LIGHT PRESSURE CHARACTERISTICS BY NUMERICAL MODELING OF THE ORBITAL SC MOTION

It is obvious that to determine the LP characteristic S, σ, or R, formulas (1), (2), or (3) can be used during numerical modeling of orbital SC motion. However, this procedure can require a very large volume of calculations. In this case, it is expedient to interpolate the results of preliminary virtual photo session.

Each of the LP characteristics is a function of longitude and latitude angles φ and ψ: $f = f(\varphi,\psi)$. We assume that as a result of virtual photo session, its discrete representation

$$f_{ij} = f(\varphi_i, \psi_j) \quad (i = 1,\ldots,N_\varphi;\, j = 1,\ldots,N_\psi)$$

is obtained on a certain uniform grid with steps $\Delta\varphi = 360°/(N_\varphi - 1)$ and $\Delta\psi = 180°/(N_\psi - 1)$. To interpolate these data, it is more convenient to use the inverse weighted distance method [6] with exponent 2.

Suppose that it is necessary to determine the characteristic for any pair of values φ and ψ. We first find the zone of influence for this pair: it is the rectangle of the grid with sizes $\Delta\varphi \times \Delta\psi$ comprising this pair with node values $f_{i,j}$, $f_{i,j+1}$, $f_{i+1,j}$, and $f_{i+1,j+1}$, where

$$i = [\varphi/\Delta\varphi] + 1, \quad j = [\psi/\Delta\psi] + 1\,.$$

Then the function can be written approximately as

$$f \approx g = \frac{w_{i,j} f_{i,j} + w_{i,j+1} f_{i,j+1} + w_{i+1,j} f_{i+1,j} + w_{i+1,j+1} f_{i+1,j+1}}{w_{i,j} + w_{i,j+1} + w_{i+1,j} + w_{i+1,j+1}}\,,$$

where $w_{i,j} = (\varphi - \varphi_i)^2 + (\psi - \psi_i)^2$.

As an example, Figs. 5 and 6 show results of interpolation by the inverse weighted distance method from the data of virtual photo session of the GLONASS SC model. The SC solar panels are oriented at an angle of 45° relative to the axis of the cylindrical SC body (see Figs. 2–4). The results were obtained on the grid of φ and ψ values with step $\Delta\varphi = \Delta\psi = 11.25°$.

It should be noted that the angles φ and ψ (the polar coordinates of the light source) are indicated in the coordinate system rigidly affixed to the SC (see Fig. 2): the abscissa coincides with the axis of the cylindrical body, the ordinate coincides with the axis of rotation of solar panels, and the *Z* axis supplements this system to the right-hand one. Therefore, when modeling the LP using the obtained characteristics, the SC orientation (rotational motion) relative to the Sun should be known.

For a model reflectance R (or other LP characteristic) depending on the satellite orientation angles φ and ψ, its average value can be easily obtained. For this purpose, it is possible to use numerical averaging of the characteristic from the data of virtual photo session:

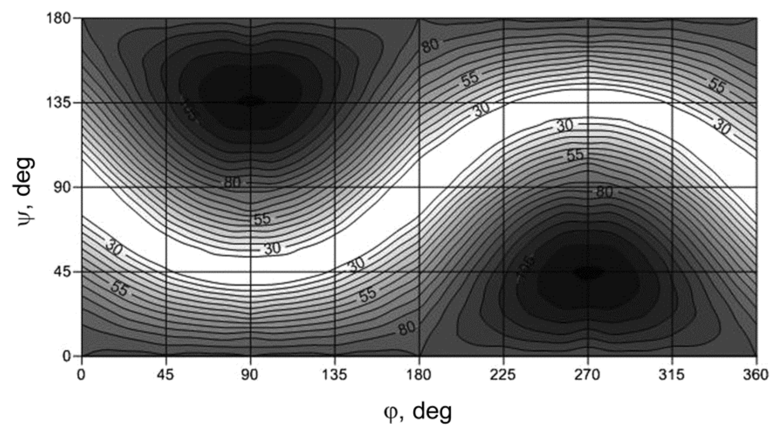

Fig. 5. Average cross sectional area of the GLONASS S satellite depending on the angles φ and ψ .

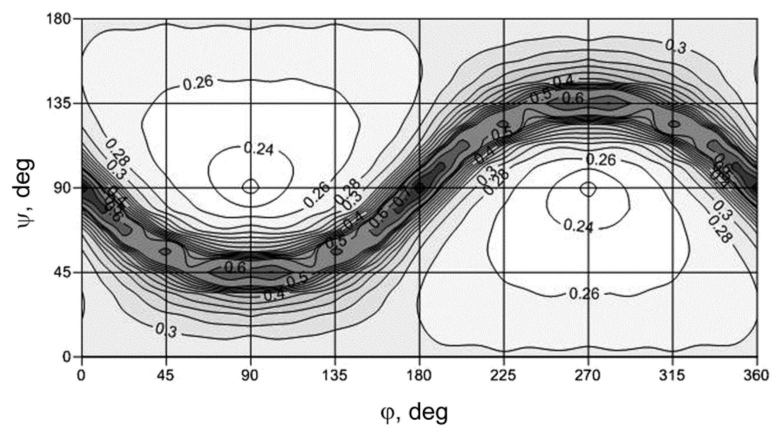

Fig. 6. Average reflection coefficient σ for the GLONASS satellite depending on the angles φ and ψ .

$$\bar{R} = \frac{1}{4\pi} \sum_{i,j} R(\varphi_i, \psi_j) \sin \psi_j \Delta\varphi \Delta\psi \ .$$

Thus, the following average values of the effective surface characteristics were obtained for the GLONASS SC:

$$\bar{R} = 17.67 \text{ m}^2, \quad \bar{\sigma} = 0.34, \quad \bar{S} = 61.92 \text{ m}^2 \ .$$

CONCLUSIONS

In this work, the method for determining the effective surface area of the spent satellite has been described depending on the spatial orientation for the point-set SC model. The method was considered on the example of the spent GLONASS spacecraft. To use the suggested model, it is necessary to know the orientation (the rotational motion) of the SC relative to the Sun. If the rotational motion is not known, the average model described in the work can be used. The presented models can be used for any objects with known configuration.

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