DOI 10.1007/s11182-017-0994-x

Russian Physics Journal, Vol. 59, No. 11, March, 2017 (Russian Original No. 11, November, 2016)

CONSTRUCTION AND IDENTIFICATION OF PROFILES OF CURVATURE RADIATION OF PULSARS

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Parameters of the observed radiation of pulsars are identified with the help of numerical simulation within the framework of the nonlinear least squares problem. With the help of the obtained parameter values, we have constructed profiles of radiation and indicatrices of the angular distribution of the instantaneous radiated power for experimentally observed pulsars.

Keywords: profile, pulsar, indicatrix, curvature radiation, method of least squares, Gauss–Newton method, numerical simulation.

INTRODUCTION

Pulsars were discovered by Miss Jocelyn S. Bell in 1967 in the Cavendish Laboratory headed by Anthony Hewish in an examination of the scintillations of quasars arising upon the passage of electromagnetic waves through the solar plasma [1]. This discovery was of fundamental importance for the development of astrophysical research into cosmic radio waves.

Pulsars have been associated with neutron stars [2] – extremely small, rapidly rotating remnants of the explosions of supernovae – and are sources of precisely periodic, pulsed radiation, whose character is explained by the lighthouse model. According to this model, the magnetic axis of the pulsar, along which the radiation passes, and the axis of rotation of the pulsar do not coincide, which leads to the result that we observe individual pulses from pulsars and not continuous radiation. If the radio beam from a pulsar never illuminates Earth, then this pulsar remains invisible for observers; therefore, not every neutron star can be observed from Earth as a pulsar.

The given work employs the universal kinematic method based on the model of Radhakrishnan and Cooke, discussed in [3], to construct periodic profiles of radiation from pulsars. According to this model, the radiation source consists of fluxes of very-fast-moving particles (jets), emanating from the magnetic poles of the pulsar in the direction of the magnetic field lines. The idea of the method proposed here consists in the following: the radiation profile of the pulsar is found as the line of intersection of the angular distribution, rotating together with the pulsar, of the instantaneous radiated power from a source located on the trajectory of the jet, with the line of sight of the observer, which is fixed in space.

1. INDICATRIX OF THE RADIATION OF AN ARBITRARY MOVING RELATIVISTIC CHARGE

The equation of the surface representing the indicatrix of the instantaneous radiated power is given by the exact theory of radiation of relativistic charged particles. The angular distribution of the instantaneous radiated power in the direction of the solid angle $d\Omega = \sin\theta \, d\theta \, d\varphi$ for an arbitrary moving charge in dimensionless form has the form [4]

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Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, No. 11, pp. 152–156, November, 2016. Original article submitted July 25, 2016.

$$\rho(\alpha,\beta,\theta,\varphi) = \frac{1}{(1-\beta\cos\theta)^3} + 2\beta\frac{\sin\theta\cos\varphi\sin\alpha + \cos\theta\cos\alpha}{(1-\beta\cos\theta)^4}\cos\alpha$$
$$+\left(1-\beta^2\right)\frac{(\sin\theta\cos\varphi\sin\alpha + \cos\theta\cos\alpha)^2}{(1-\beta\cos\theta)^5}, \tag{1}$$

where $\beta = u/c$ is the velocity of the radiating particles, expressed in units of the speed of light, α is the angle between the velocity and the acceleration, and θ and φ are the angles of the spherical coordinate system. The expression is represented in a form suitable for the construction of profiles of the instantaneous radiation. If we interpret it as a radius vector, then it will describe some surface in the spherical coordinate system, which we call the indicatrix of radiation. The result of the intersection of the line of sight of the observer with the indicatrix is the radiation profile of the pulsar, or the time sweep of the radiated power.

In essence, this is an inverse problem, whose solution does not initially depend on this or that model of the magnetosphere of the pulsars and can help us find the actually existing configuration of locations of radiation sources in experimentally observed pulsars. To identify the profiles of polarized radiation of pulsars constructed in this way with experimentally observed profiles assumes the use of an entire set of constant parameters, such as the inclination angle of the magnetic axis of the pulsar relative to its rotation axis, and also the angles formed by the line of sight with the rotation axis of the pulsar and with the direction of its magnetic axis. In addition, the given model allows one to vary the parameters of the radiation itself, for example, the number and energy of the radiating particles (the gamma factor), the magnetic field strength, and the radius of curvature of the trajectory.

2. METHOD OF CONSTRUCTING PROFILES OF RADIATION FROM PULSARS

Figure 1 depicts the coordinate system of a pulsar together with a jet of moving particles. Since the pulsar is located a great distance from the Earth, it is depicted as a point located at the origin. The unit vector of a light ray incident on the observer is denoted as $\boldsymbol{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$. The axis of rotation is assigned by the vector $\boldsymbol{s} = (0,0,1)$. The direction of the magnetic field $\boldsymbol{\mu}$ is defined by the unit vector $\boldsymbol{k}$, which rotates about the $\boldsymbol{s}$ axis with a constant inclination angle η.

A ray of light arrives at the observer from the indicatrix of the angular distribution of the radiated power. The indicatrix is initially defined in the unprimed coordinate system *XYZ* with the help of the spherical angles θ and φ with symmetry plane *XZ* in which the jet is located. The plane of the jet does not have to coincide with the plane of rotation of the magnetic axis of the pulsar $\boldsymbol{k}, \boldsymbol{s}$. In each specific case, the plane of the trajectory of the jet can be rotated by some constant angle λ about the magnetic axis, where this angle is determined individually for each pulsar.

The expression from the preceding paragraph allows us to construct a profile of pulsars for which the trajectory of radiation of the charged particles at the moment of emission is located in the *XY* plane, i.e., when $\beta \| Z$. In order to construct indicatrices of the total power of instantaneous radiation for an arbitrary orientation of the velocity and acceleration of the particle relative to the coordinate axes at the moment of emission, it is necessary to transform to the pulsar–observer coordinate system after making two left rotations: with respect to the *Y* axis by the angle η and with respect to the *Z* axis by the angle λ. As a result, for these angles we obtain the following transformations (Fig. 1):

$$\sin\theta\cos\varphi = \left[\sin\zeta\cos(\Omega t - \delta)\cos\eta - \cos\zeta\sin\eta\right]\cos\lambda + \sin\zeta\sin(\Omega t - \delta)\sin\lambda, \tag{2}$$

$$\sin\theta\sin\varphi = -\left[\sin\zeta\cos(\Omega t - \delta)\cos\eta - \cos\zeta\sin\eta\right]\sin\lambda + \sin\zeta\sin(\Omega t - \delta)\cos\lambda,$$

$$\cos\theta = \sin\zeta\cos(\Omega t - \delta)\sin\eta + \cos\zeta\cos\eta.$$

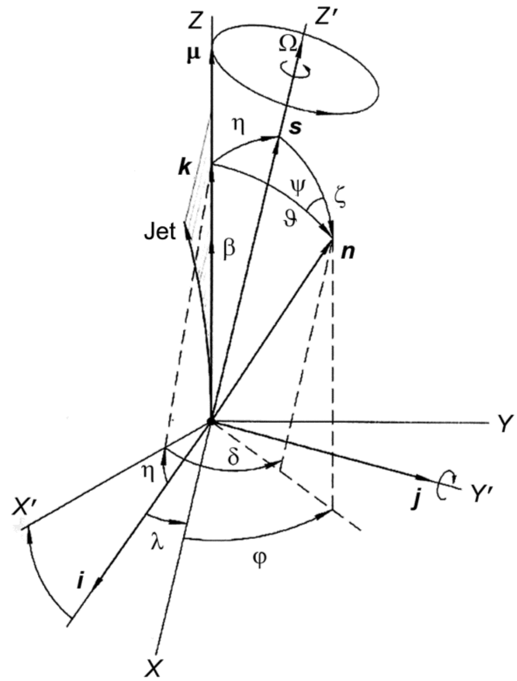

Fig. 1. The observer–pulsar coordinate system.

Here it has been taken into account that the solid angle $\mathrm{d}\Omega = \sin\theta \mathrm{d}\theta \mathrm{d}\varphi$ is invariant under rotations of the coordinate system.

3. DETERMINATION OF THE PARAMETERS OF THE PROFILES OF RADIATION FROM PULSARS FROM OBSERVATIONS

The parameters of the kinematic model were determined from the observed values of the instantaneous radiated power [5] within the framework of the nonlinear problem of least squares, which reduces to minimization of the functional $S(\boldsymbol{q}) = \| \rho^O - \rho^C(\boldsymbol{q}) \|^2 \to \min$. Here $\boldsymbol{\rho}^O = (\rho_1^O, \ldots, \rho_N^O)^T$ and $\boldsymbol{\rho}^C = (\rho_1^C, \ldots, \rho_N^C)^T$ are N-dimensional vectors of the measurements and their model representations (1) with allowance for the transformation of coordinates (2) at the times $t_1, \ldots, t_N$, corresponding to the angular values $\varphi'_1, \ldots, \varphi'_N$, N is the number of measurements, and $\boldsymbol{q} = (\alpha, \beta, \lambda, \eta, \theta)^T$.

Since the problem is nonlinear, it is solved numerically by the Gauss–Newton iterative method (when needed, with damping) [6]. According to this method, at each iteration a correction $\Delta\boldsymbol{q} = -h\boldsymbol{Q}^{-1}\boldsymbol{G}$ is added to the parameters to be determined, $\boldsymbol{q}$, where $\boldsymbol{Q} = \boldsymbol{A}^T\boldsymbol{A}$, $\boldsymbol{A} = \partial\boldsymbol{\rho} / \partial\boldsymbol{q}$, $\boldsymbol{G} = -\boldsymbol{A}^T\boldsymbol{B} = \partial S / \partial\boldsymbol{q}$, $\boldsymbol{B} = \boldsymbol{\rho}^O - \boldsymbol{\rho}^C$, and $h \le 1$ is the damping factor. Here damping is applied when the corrections to the angular parameters become greater than 0.01 (radians), and for the parameter β, when they become greater than 0.001. The iterative process is terminated when $\| \boldsymbol{q} \| < 10^{-8}$.

4. COMPARISON WITH EXPERIMENTAL DATA ON THE PROFILES OF PULSARS

To confirm the expounded theory, we obtained the parameters of the equation for the indicatrix for a series of observed pulsars and constructed theoretical profiles of their radiation, which gave good agreement with the experimentally obtained values. Figures 2 and 3 plot the experimental data (Figs. 2*a* and *b* and Figs. 3*a* and *b*) [4] and

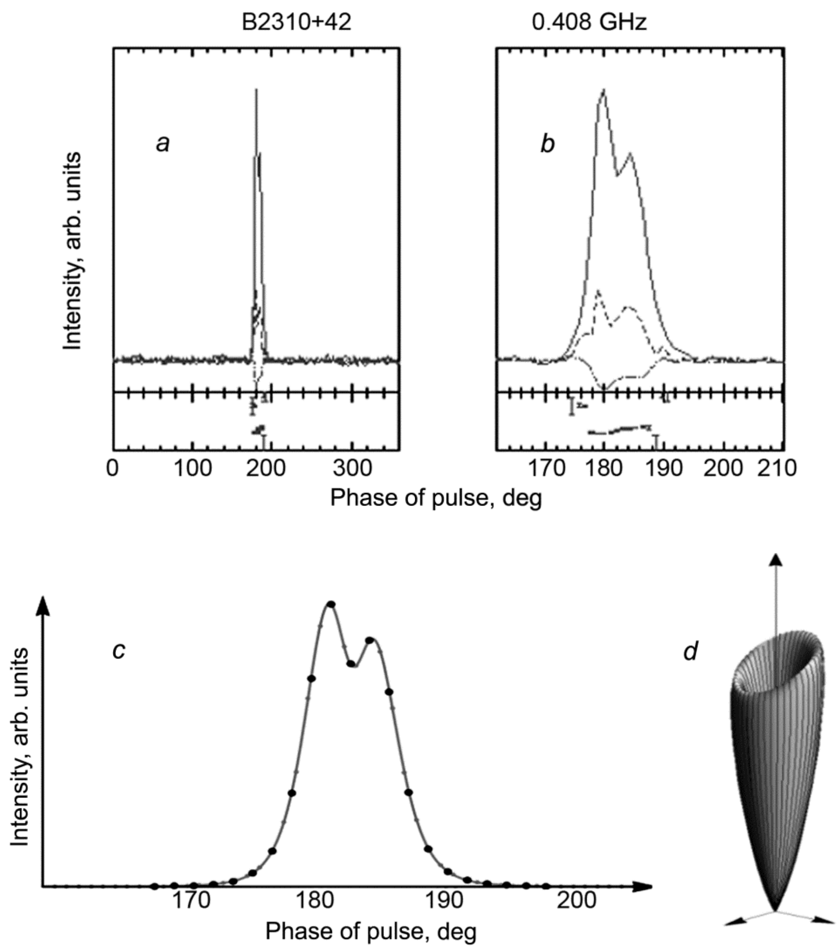

Fig. 2. PSR 2310+42: *a* and *b* show an experimental average profile, *c* compares the theoretical profile of the pulsar (solid curve) with the experimentally observed profile (points), and *d* displays the indicatrix of the pulsar constructed for the obtained set of parameters.

compare the theoretically constructed profile with the experimental data (Figs. 2*c* and 3*c*) and display indicatrices for the corresponding neutron star parameters (Figs. 2*d* and 3*d*). The graphs plot the normalized distribution of the total radiated power along the vertical axis versus angle for an arbitrarily moving relativistic charge. As the objects for which we calculated the radiation parameters, we took the pulsars PRS 2310+42 (kinematic model parameters determined: $\beta = 0.98999986$, $\alpha = 24.99721365°$, $\lambda = 60.0102946°$, $\eta = 12.0001117°$, and $\theta = 15.0000547°$) and PRS 0655+64 (kinematic model parameters determined: $\beta = 0.990000003$, $\alpha = 9.990358837°$, $\lambda = 60.1069155898°$, $\eta = 14.00028°$, and $\theta = 11.00007248°$). In the determination of the pulsar parameters, the iterative process converged after not more than 20 steps for each pulsar for a good initial approximation of the parameters.

CONCLUSIONS

The method of kinematic construction of pulsar radiation profiles developed in this work for instantaneous curvature radiation from the magnetic poles, based on the corresponding indicatrices of synchrotron radiation (the Radhakrishnan and Cooke model), gives good agreement with the experimentally observed radiation arriving from

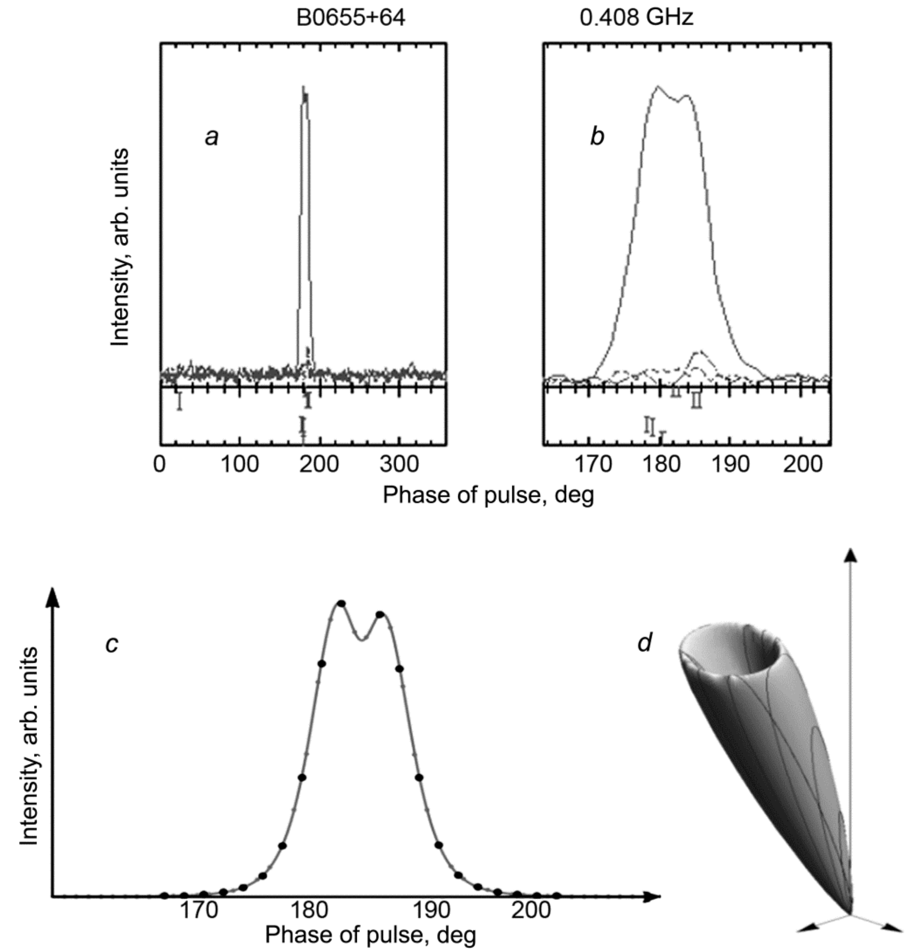

Fig. 3. The same as in Fig. 2, but for PRS 0655+64.

some specific pulsars. A virtue of the method is that it lays out a set of parameters that influence the formation of the pulsar radiation profiles: the spherical angles of the observer, the angular velocity and period of rotation of the pulsar, the inclination angle of the magnetic axis of the pulsar and the inclination angle of the line of sight of the observer relative to the angular velocity vector, the inclination angle of the indicatrix of radiation relative to the magnetic axis of the pulsar, and the angle of rotation of the symmetry plane of the indicatrix itself, and finally, the kinematic characteristics of the radiating particles, such as their velocity and acceleration vectors. All of these parameters have allowed us with a high degree of accuracy to reproduce the radiation profiles of experimentally observed pulsars, which has been demonstrated in examples of an entire series of specific pulsars. The accuracy in determining the kinematic parameters for the observed profile was achieved with the help of numerical simulation of the indicatrix parameters using the Gauss–Newton method. Thus, it is hoped that the identification of pulsar radiation parameters performed in the given work will enable a deeper understanding of the phenomenon of periodicity of radiation of neutron stars.

REFERENCES

1. I. S. Shklovskii, Stars: Their Birth, Life, and Death, W. H. Freeman, New York (1978).
2. T. Gold, Nature, **218**, 731 (1968).

3. V. A. Bodovitsyn and E. A. Nemchenko, in: Proc. All-Russian Astrophysical Conf. “Many Faces of the Universe VAK-2013”, Saint Petersburg (2013), pp. 35–36.
4. V. G. Bagrov and V. A. Bordovitsyn, in: Theory of Radiation of Relativistic Particles [in Russian], V. A. Bordovitsyn, ed., Fizmatlit, Moscow (2002), pp. 13–80.
5. European Pulsar Network: Data Achieve Browsing Facility, http://www.jb.man.ac.uk/research/pulsar/Resources/epn/browser.html (April 3, 2016).
6. J. Ortega and W. Reinboldt, Iterative Solution of Nonlinear Equations in Several Variables, Academic Press, Philadelphia (2000).